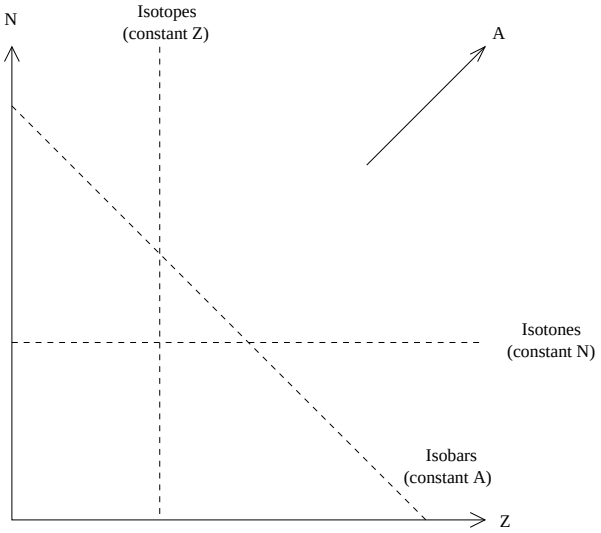
Nuclear and Particle Physics - Lecture 18
The nuclear force

1 Introduction

Protons and neutrons can actually bind together through the strong force strongly enough to form bound states. These can alternatively be thought of as bound states of 6, 9, 12... quarks. These bound states form the nuclei of atoms and the sum of the proton charges is what holds the outer atomic electrons in their orbits.

2 Nuclei

A nucleus is described by the number of protons, Z, and the number of neutrons, N. The total number of protons and neutrons (collectively called nucleons) is denoted by the atomic mass number A which is by definition $Z + N$. Some nomenclature is that nuclei with the same atomic number Z but differing values of N are called *isotopes* of each other. Because they have the same nuclear charge, they tend to be very similar chemically, although the nuclear properties themselves are very different. Less often used terms are that nuclei with the same N but differing Z are called *isotones* and nuclei with the same A but differing Z (and hence N) are called *isobars*.



You might think that if the baryons (and mesons) are colourless, i.e. have no net colour charge, then there would be no strong force between them. This is not actually true; the same argument in terms of EM charge would argue that there would be no force between neutral atoms. This is clearly wrong; if true all matter would be an ideal gas and no liquids or solids could form.

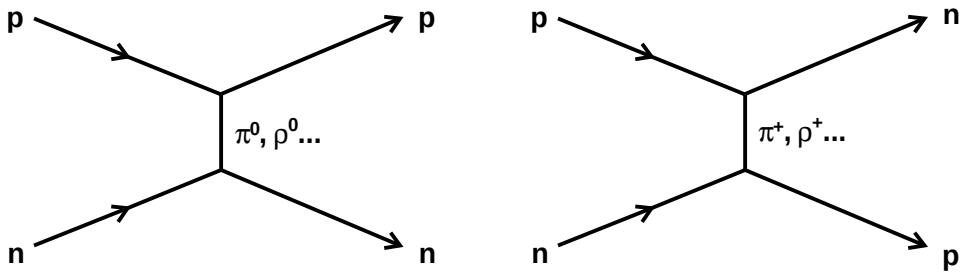
3 Nucleon forces

In the case of atoms, although they have no charge, there are residual "multipole" electric fields which can interact although these potentials fall off a lot quicker than $1/r$. These forces are normally refered to a Van de Waals forces and are often parametrised by a potential of the form $a/r^{12} - b/r^6$. There is a similar effect for baryons; there is a residual Van de Waals-like strong force between them which, although much weaker than the full strong force, is still able

to overcome the proton EM repulsion and bind nucleons together into nuclei. We will call this force the nuclear force. Like the Van de Waals force, it falls off quickly with r and so is short range. Also like the Van de Waals force, it becomes large and repulsive at very short distances.

One important property of the nuclear force is that it is approximately independent of the nucleons involved. Both the proton and neutron and made of three quarks with very small masses compared to the mass of the QCD field surrounding them, which results in the proton and neutron masses being very close; $m_p = 938.3$ MeV/c^2 and $m_n = 939.6$ MeV/c^2, which only differ by 0.1%. All the quarks have the same strong charge and are in equivalent wavefunctions, so the residual force should be the same for both. Clearly, there is a repulsive EM force between protons which is absent for neutrons, but the nuclear force is effectively the same for both types of nucleon.

There is another physical picture of the same residual nuclear force in terms of boson exchange, which then makes it more like our other forces. The obvious question is; what bosons are exchanged? They cannot be gluons as that would leave net colour charge on the radiating and absorbing nuclei. In fact, the nuclear force can be well described by meson exchange, i.e. radiation and absorption of pions, rhos, etc.



Since for any force, the range $\sim \hbar/mc$, then for all but the shortest distances, pion exchange dominates as the pions are so much lighter than the other mesons. Since $m_\pi \sim 140$ MeV/c^2, then the range is $\hbar/m_\pi c = 1.4$ fm. This again shows the nuclear force is very short range and now also sets a scale for this range. This picture is most clearly needed for the case of proton-neutron elastic scattering. As might be expected, the scattering is mainly peaked forward, meaning small angles are most likely; this is similar to $e\mu$ scattering. However, there is also a large peak at very large angles near 180°; this is easy to understand in terms of the above charged pion exchange diagram, where the kinematics are very similar to neutral pion exchange but the proton and neutron appear to swap direction.

Both pictures of the nuclear force are valid in certain ways and both help to understand aspects of its characteristics. We will use these concepts to try and understand how the nucleons bind together into nuclei. First, we shall look at a few simple nuclei and then at the behaviour of larger nuclei with many nucleons. For the latter, we shall try to quantitatively justify the binding energy B_E, for which

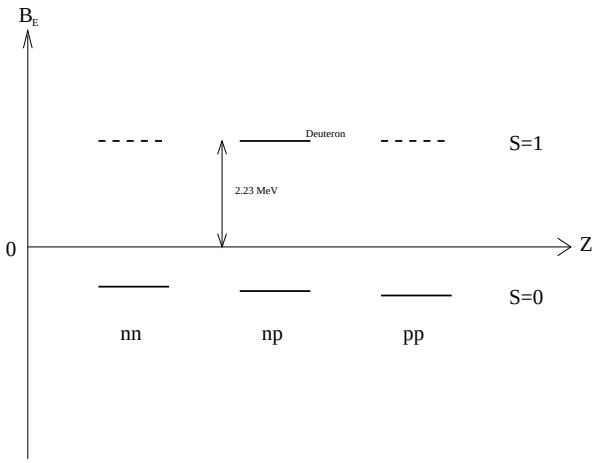
$$m_{\text{Nucleus}} = Zm_p + Nm_n - \frac{B_E}{c^2}$$

It is obvious that for a nucleus to be stable, then $B_E > 0$.

4 Simple nuclei

The simplest nucleus besides a single nucleon is the deuteron, with $Z = N = 1$, i.e. one proton and one neutron. This is written as $^2_1$H as it is an isotope of hydrogen (and when made into water,

makes it "heavy water"). If the force between nucleons is independent of nucleon type, then we might expect there to be *pp* and *nn* nuclei as well. However, neither of these combinations is stable, i.e. they do not have positive binding energy. This is because, in these cases, the two nucleons are identical and the Pauli exclusion principle comes into play. In fact, the deuteron has spin 1 which is formed from the two spin 1/2 nucleons in an $S = 1$ state. This lowest state has $L = 0$ as is usually the case. The next highest state is $S = 0$, again with $L = 0$, so these states are split by a hyperfine interaction, just like for the mesons; however, in the meson case, the $S = 0$ was lower. But, as we have seen several times, the $S = 1$ state formed from two spin 1/2 particles is symmetric under interchange of the fermions. There is no colour part of the wavefunction now to help out, so this state cannot be occupied by identical particles. Hence, although the force between the nucleons is the same, the *pp* and *nn* states are forbidden and so don't exist. For the higher $S = 0$ state, where the fermions are antisymmetric, all three combinations of *pp*, *pn* and *nn* are found (where the *pn* state is the first excited state of the deuteron) but this state has negative binding energy and is not stable.

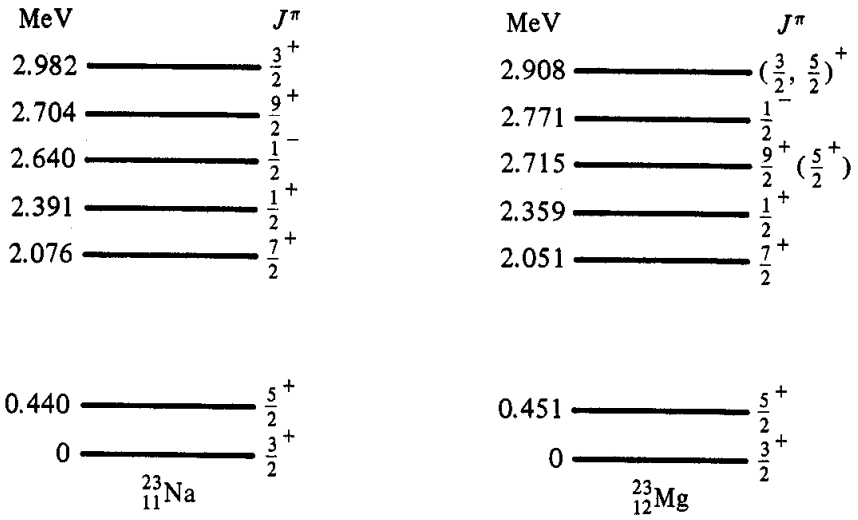


The deuteron itself actually has quite a low binding energy $E_B = 2.23$ MeV, i.e. 1.1 MeV per nucleon, which, as we will see, is quite small compared with the average value of around 8 MeV per nucleon for most other nuclei.

Similarly, going up a nucleon, it turns out there are two bound states of three nucleons; tritium, ${}^{3}_{1}\mathrm{H}$, and helium-3, ${}^{3}_{2}\mathrm{He}$, which are states of two of one nucleon and one of the other with equivalent wavefunctions. (Tritium in fact beta decays to helium-3.) There is no bound state of *ppp* or *nnn* as Pauli exclusion forbids these combinations in the tritium and helium-3 wavefunction. There is again a wavefunction with a higher energy which allows these, as well as excited tritium and helium-3 states but, as for the deuteron, these are unbound.

For four nucleons, helium-4, ${}^{4}_{2}\mathrm{He}$, also called the alpha particle, is extremely stable with $B_E = 28.3$ MeV, i.e. 7.1 MeV per nucleon. It is spin 0 and has the two protons and two nucleons all packed close together in a low energy level. There are no other bound nuclei for $A = 4$.

We can check the nuclear force is independent of the nucleons involved most easily by comparing nuclei where all the protons and neutrons are interchanged, as there are then no differences due to the Pauli exclusion principle. Such nuclei are called "mirror nuclei"; tritium and helium-3 are one example. Another example, specifically the excited states of ${}^{23}_{11}\mathrm{Na}$ (which has $Z = 11$ and $N = 12$) and ${}^{23}_{12}\mathrm{Mg}$ (which has $Z = 12$ and $N = 11$), is shown below.

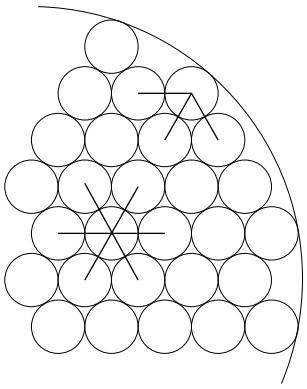


Hence, the levels are seen to be very similar, with the differences explainable in terms of the EM force on the protons due to their charge.

Hence, what we find is that the strong force between protons and neutrons seems to be equal but that the Pauli exclusion principle is very important for determining which states are allowed and indeed tends to favour states with approximately equal numbers of protons and neutrons, i.e. $Z \approx N$. We will now look at the properties of nuclei with many nucleons, i.e. nucleons "in bulk".

5 The liquid drop model

We have seen that the nuclear force has a very short range ~ 1 fm, which is only the size of a nucleon radius. It also saturates, i.e. becomes very large and repulsive for short distances. This means that in a nucleus with many nucleons, they will not all crowd together at the origin but will spread out to occupy a finite volume each, liked packed spheres. In addition, the force each then imposes on the others is so short range that it is negligible for all but the nearest neighbours.



This is physically similar to the saturation in a water drop, where the Van de Waals forces between the water molecules effective make only the nearest neighbour interaction significant.

Because of this analogy, the resulting nuclear model is often called the "liquid drop" model. It gives a good description of the binding energies of the ground states of nuclei with many nucleons, where many in practise means $A \geq 20$.

There are two immediate consequences of this model. Firstly, the volume of the nucleus clearly would be expected to increase linearly with the number of nucleons, so $V \propto A = N + Z$. Since $V = 4\pi r^3/3$, the nuclear radius r should go as $r \propto A^{1/3}$. This is a good approximation and is usually expressed as $r = r_0 A^{1/3}$, where $r_0 = 1.2$ fm, roughly the radius of a nucleon. The second consequence is that binding energy for each nucleon would be expected to be constant. This is because each nucleon only binds to its nearest neighbours, so the contribution to the binding energy from each is a fixed value. Hence, we would expect $B_E \propto A$, which is again found to be roughly true; as stated above, the binding energy for heavy nuclei is found to be roughly 8 MeV per nucleon. However, there is an obvious problem with this approximation and that is the nucleons on the surface of the nucleus. These will not have as many nearest neighbour bonds as the ones well within the nuclear volume as there are no nearest neighbours outside the nucleus. This is exactly the same effect which gives rise to surface tension in a water drop, so again the analogy holds. We would therefore expect that there will be a correction to the binding energy proportional to the sphere surface area $4\pi r^2$. Since $r \propto A^{1/3}$, then our expression for the binding energy becomes

$$B_E = a_v A - a_s A^{2/3}$$

where a_v (the volume term constant) and a_s (the surface term constant) are parameters to be determined from data.